4.4c $c(t) = (t-5)^3 \ln(t)$

Find critical pts & classify.

$t \in (0,\infty)$

$$c'(t) = 3(t-5)^2 \ln t + \frac{(t-5)^3}{t}$$

$$= (t-5)^2 \left[3\ln t + \frac{(t-5)}{t} \right]$$

$$= 0$$

$$\iff t = 5 \text{ or } 3\ln t + \frac{(t-5)}{t} = 0$$

Type some Sage code below and

```
f(x)= 3*ln(x)+(x-5)/x
find_root(f(x),.1,5)
```

Evaluate

```
1.80450000049554678
```

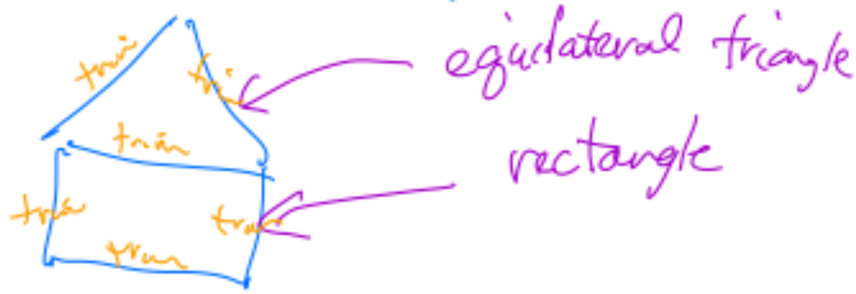
$t \approx 1.805$

$c'(t) = \; - \; - \; - \; 0 \; + \; + \; + \; + \; 0 \; + \; + \; + \; +$

t: 0, 1.805, 5

More examples of max/min problems (optimization).

Ex Ceu wants to design a window like this

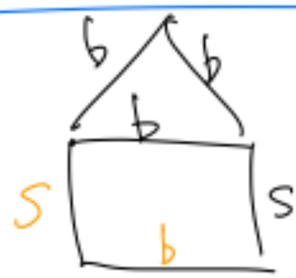


That has 20 sq ft of window. How should it be designed so

(a) It has the minimum amount of window trim?

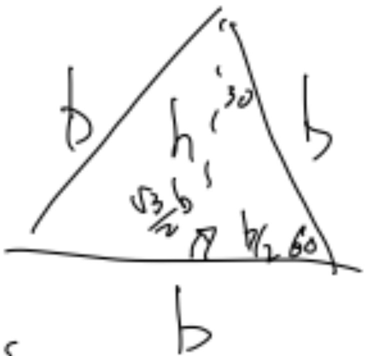
(b) So that it has the maximum amount of window trim?

Solution:

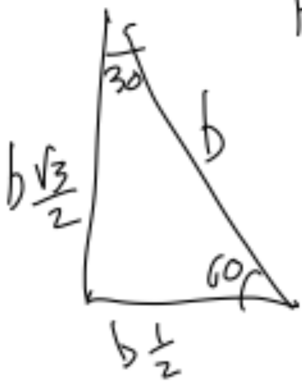


Function (quantity we want to minimize or maximize) $P = 4b + 2s$

$$20\,sq\,ft = Area = \left(\frac{1}{2}bh\right) + bs$$



$$\rightarrow h = \frac{\sqrt{3}}{2}b$$



$$\Rightarrow Area = 20\ ft^2 = \frac{1}{2}b\left(\frac{\sqrt{3}}{2}\right)b + bs$$

$$\Rightarrow 20 = \frac{\sqrt{3}}{4}b^2 + bs$$

we can subst. for one variable so P only has 1 variable

$$\Rightarrow bs = 20 - \frac{\sqrt{3}}{4}b^2$$

$$s = \frac{20}{b} - \frac{\sqrt{3}}{4}b$$

$$\Rightarrow P(b) = 4b + 2s$$

$$= 4b + 2\left(\frac{20}{b} - \frac{\sqrt{3}}{4}b\right)$$

$$\Rightarrow P(b) = 4b + \frac{40}{b} - \frac{\sqrt{3}}{2}b$$

$$\Rightarrow \boxed{P(b) = \left(4 - \frac{\sqrt{3}}{2}\right) b + \frac{40}{b}}$$

function to minimize for (a)
maximize for (b).

Interval: $0 < b \leq$?

max b is when $\frac{1}{2} bh = 20$

$$\Rightarrow \frac{\sqrt{3}}{4} b^2 = 20$$

$$b^2 = \frac{80}{\sqrt{3}}$$

$$b = \frac{\sqrt{80}}{\sqrt[4]{3}}$$

max $b = 6.796$

Interval $0 < b \leq 6.796$

$$P(b) = \left(4 - \frac{\sqrt{3}}{2}\right) b + \frac{40}{b}$$

$$P'(b) = \left(4 - \frac{\sqrt{3}}{2}\right) + \frac{-40}{b^2} = 0$$

$$\left(4 - \frac{\sqrt{3}}{2}\right) = \frac{40}{b^2}$$

$$b^2\left(4 - \frac{\sqrt{3}}{2}\right) = 40$$

$$b^2 = \frac{40}{\left(4 - \frac{\sqrt{3}}{2}\right)}$$

$$b = \sqrt{\frac{40}{4 - \frac{\sqrt{3}}{2}}} \qquad b > 0$$

$$\boxed{b \approx 3.57 \text{ ft.}}$$

only cr. pt in interval

$(0, 6.796]$ $\qquad$ $b = 6.796$ possibility

$$\lim_{b \to 0^+} P(b) = \lim_{b \to 0^+} \left[\underbrace{\left(4 - \frac{\sqrt{3}}{2}\right)b}_{\to 0} + \underbrace{\frac{40}{b}}_{\to +\infty}\right] = +\infty$$

(b) $\boxed{\text{No global max}}$

(keep making b very small).

$P'(b)$: $\quad - \; - \; - \quad 0 \quad + \; + \; + \; + \; +$

b: $\quad 0 \qquad\qquad 3.57 \qquad\qquad 6.796$

3.57 local min.

b	$P(b)$

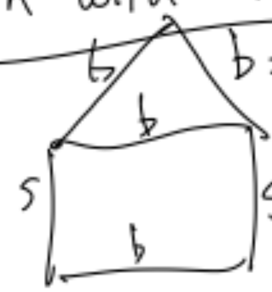
Only cr pt is local min
$\Rightarrow$ global min

$\therefore$ $b = 3.57$ is minimum.

$$s = \frac{20}{b} - \frac{\sqrt{3}}{4} b = \frac{20}{3.57} - \frac{\sqrt{3}}{4}(3.57)$$

$$= 4.06$$

(a) $\therefore$ Design with minimum trim is

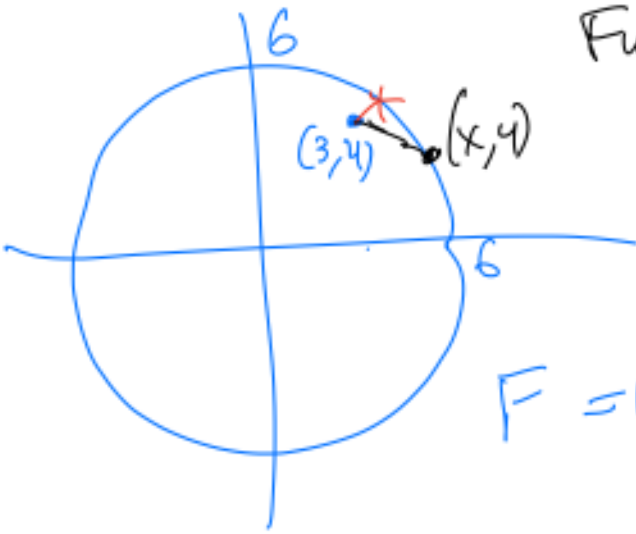


$b = 3.57$ ft.

$s = 4.06$ ft

Ex

Find the point on the graph of $x^2 + y^2 = 36$ that is closest to $(3, 4)$.



Function = $\begin{pmatrix}\text{distance from} \\ (x,y) \text{ to } (3,4)\end{pmatrix}^2$

Want to minimize this.

$F = (x-3)^2 + (y-4)^2$

$x^2 + y^2 = 36$

$x^2 = 36 - y^2$

$x = \pm\sqrt{36 - y^2}$ (we choose the + one (see picture))

$x = \sqrt{36 - y^2}$

$F(y) = \left(\sqrt{36-y^2} - 3\right)^2 + (y-4)^2$

Interval: $0 \le y \le 6$.

$[0,6]$ closed. Abs max or min must be $y=0$, $y=6$, or $y = $ cr pt.

$$F'(y) = 2\left(\sqrt{36-y^2}-3\right)\cdot\left[\tfrac{1}{2}(36-y^2)^{-1/2}\cdot(-2y)\right] + 2(y-4)$$

$$F'(y) = -2\left(\sqrt{36-y^2}-3\right)\left(\frac{y}{\sqrt{36-y^2}}\right) + 2(y-4)$$

$$= \left(-2\sqrt{36-y^2}+6\right)\left(\frac{y}{\sqrt{36-y^2}}\right) + 2(y-4)$$

$$= \frac{-2y\sqrt{36-y^2}}{\sqrt{36-y^2}} + \frac{6y}{\sqrt{36-y^2}} + 2(y-4)$$

$$= \cancel{-2y} + \frac{6y}{\sqrt{36-y^2}} + \cancel{2y} - 8$$

$$F'(y) = \frac{6y}{\sqrt{36-y^2}} - 8 = 0 \text{ or undefined}$$

$y=6$ undefined. ✓

$$\frac{6y}{\sqrt{36-y^2}} = 8$$

$$6y = 8\sqrt{36-y^2} \Rightarrow 36y^2 = 64(36-y^2)$$

$$\Rightarrow 36y^2 = 2304 - 64y^2$$

$$\Rightarrow 100y^2 = 2304 \Rightarrow y^2 = 23.04$$

$\Rightarrow y = \sqrt{23.04} = 4.8$. only 1 cr pt.

y	$f(y) = \left(\sqrt{36-y^2} - 3\right)^2 + (y-4)^2$
0	$(6-3)^2 + 4^2 = 3^2 + 4^2 = 25$
4.8	$\left(\sqrt{36-23.04} - 3\right)^2 + (.8)^2 = 1 \longleftarrow$ global min.
6	$(3)^2 + (2)^2 = 13.$

$y = 4.8$ global min.

$x = \sqrt{36 - (4.8)^2} = 3.6$

$$\boxed{(x,y) = (3.6, 4.8)}$$